Exercise set I

Subject: Topology I

(1) Show that the space $\mathbb{R}^2/\mathbb{Z}^2$ is homeomorphic to the space $S^1 \times S^1$.
(2) Find an action of C_2 on the torus whose orbit space is homeomorphic to a cylinder.
(3) Describe the orbit space of the action of $SO(n)$ on $\mathbb{R}^n$.
(4) With respect to the usual inclusion $\mathbb{C}P^{n-1}$ in $\mathbb{C}P^n$, prove that $\mathbb{C}P^n/\mathbb{C}P^{n-1}$ is homeomorphic to S^{2n}.
(5) Write down a S^1-action on $\mathbb{R}P^{2n+1}$ whose quotient space is homeomorphic to $\mathbb{C}P^n$.
(6) Prove that $\mathbb{R}P^1$ is homeomorphic to S^1. Prove that $\mathbb{C}P^1$ is homeomorphic to S^2.
(7) Prove that $S^2 \times S^2$ is obtained by attaching a 4-cell to $S^2 \vee S^2$.
(8) Suppose X is a topological space. We define X_+ to be the based space whose underlying space is $X \sqcup *$ and $*$ is the basepoint. Prove that $S^n_+ \wedge S^1$ is homeomorphic to the space obtained by identifying the north and the south poles of S^{n+1}.
(9) Prove that $C(\Delta^n)$ is homeomorphic to Δ^{n+1}.