

Assignment - 2

1. Determine the conjugacy classes in A_4 .
Compute $Z(A_n)$, $n \geq 4$. (1)
2. Prove that the 3-cycles in A_5 form a single conjugacy class. Find two 5-cycles in A_5 that are not conjugate in A_5 . (1)
3. Using the description of D_n (dihedral group of order $2n$) in terms of a rotation r of order n & a reflection s of order 2, compute $Z(D_n)$, distinguishing the cases when n is odd & when n is even. (1)
4. Let $m \geq 2$. ^{Verify that} The $4m$ -element set $D = \{e, \alpha, \dots, \alpha^{2m-1}, y, \alpha y, \dots, \alpha^{2m-1} y\}$ with products given by:

$$\alpha e = \alpha = e \alpha, \quad e y = y = y e,$$

$$\alpha^i \alpha^j = \alpha^{i+j}, \quad \alpha^i (\alpha^j y) = \alpha^{i+j} y, \quad (\alpha^i y) \alpha^j = \alpha^{i-j} y,$$

$$(\alpha^i y) (\alpha^j y) = \alpha^{i-j+m}, \quad 0 \leq i, j \leq 2m-1,$$
 is a group, where powers of α are read mod $2m$.
 D is the quaternion group when $m=2$. D is called the dicyclic group of order $4m$. (1)